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THREE CONCEPTS OF PRESSURE DISTRIBUTION IN THE FIELD OF AN ACOUSTIC EMITTER IN SYSTEMS OF CONTROL OF GAS–LIQUID FLOWS

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The structure of the field used in ultrasonic control of gas–liquid flows of a short-wave acoustic emitter (AE) of continuous harmonic oscillations and long pulses is considered. The formula for emission pressure in the paraxial domain of the field is verified.

Ultrasonic methods of control of physical parameters [1–7], velocity, volumetric and mass flow rate [7–14], pressure, temperature, composition, and properties [15–21] of gas–liquid flows and parameters of technological processes [22–26] are widely used in industry and scientific research. In this case, use is often made of shortwave acoustic emitters of continuous harmonic oscillations and long pulses (or sets) of oscillations.

The field of the acoustic emitter is described by a system of wave equations. According to the data of [27], a general solution of them for the elastic potential φ of the field was first obtained by Helmholtz, who used the integral Green theorem in the function of the potential ψ of the point source:

$$\psi = r^{-1} \exp(-ikr) . \tag{1}$$

This solution of the system of equations for the potential φ (and pressure P corresponding to it) is a combination of effects distributed over an oscillating surface S of the emitter: point sources (monopoles) with the potential ψ and dipole sources with the potential φ_s of the vibrational velocity of the surface S.

In practice, three concepts of pressure distribution are used, representing it as a function of only monopoles (1), only dipoles (2), and a combination of monopoles and dipoles (3).

1. According to the first concept [27], the pressure is determined by the Rayleigh integral:

$$P = \frac{\rho c}{2\pi} \iint_S ik\psi v_n dS . \tag{2}$$

This integral was derived as early as 1954 by Skudrzyk [28] and in 1959 by Rzhevkin as a simplified solution of the system of wave equations on the basis of the Green theorem. Integration of (2) yields the Shtentsel' formula for pressure $P(z)$ on the axis of a shortwave acoustic emitter (which is still used today):

$$P(z) = P_0 [\exp(-ikz) - \exp(-ikr_m)] \exp(i\omega t) , \tag{3}$$

where

$$r_m = (z^2 + a^2)^{0.5} \tag{4}$$

is the distance from the point of receipt on the axis of the acoustic emitter to its end and

$$P_0 \exp(i\omega t) = \rho c v_n \tag{5}$$

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is the initial pressure of emission of continuous oscillations or long (compared to the period of oscillations) pulses of oscillations by the shortwave ($ka >> 1$) acoustic emitter.

Expression (3) can be written in the following form:

$$P(z) = P_0 [1 - \exp(-i\varphi_0)] \exp i(\omega t - kz) , \tag{6}$$

where the pressure phase φ_0 is determined by the expression

$$\varphi_0 = k(r_m - z) . \tag{7}$$

From integration of (2) follows the known Shokh formula [28] for pressure at the point lying at a distance q from the axis of the acoustic emitter and a distance z from the oscillating surface of the acoustic emitter:

$$P = P_s [\exp(-ikz) - J_0(\varepsilon_q) \exp(-ikr_m)] , \tag{8}$$

where

$$P_s = P_0 \exp(i\omega t) ; \quad \varepsilon_q = qka/r_m . \tag{9}$$

2. The second concept is based on the presentation of the potential φ of the field (or pressure) as a function of only dipoles. This dependence is derived by Skudrzyk from a general solution of the system of wave equations (and also on the basis of the Green theorem) in the form of the integral expression

$$\varphi = -\frac{1}{2\pi} \iint_S \varphi_s z r^{-1} \, dS \frac{\partial \psi}{\partial r} ,$$

where z/r is the characteristic of the dipole direction. By virtue of the identity of pressure (with a factor accuracy) to the potential of vibrational velocity, from this integral we have

$$P = -\frac{z}{2\pi} \iint_S P_s r^{-1} \frac{\partial \psi}{\partial r} \, dS . \tag{10}$$

To derive the expression for pressure on the axis of the piston acoustic emitter, we place the origin of coordinates (ρ_a, z, α) at the center of the acoustic emitter. In this case, $r^2 = z^2 + \rho_a^2$. Differentiation of this expression gives $rdr = \rho_a d\rho_a$ and

$$dS = \rho_a d\rho_a d\alpha = r dr d\alpha . \tag{11}$$

The limits of variation of independent variables are from 0 to 2π for α and from z to r_m for r. With this in mind, expression (10) is written as

$$P(z) = -\frac{z}{2\pi} \int_z^{r_m} P_s \frac{\partial \psi}{\partial r} dr \int_0^{2\pi} d\alpha$$

or, using values of P_s from (9),

$$P(z) = P_0 \left[1 - \frac{z}{r_m} \exp(-i\varphi_0) \right] \exp i(\omega t - kz) . \tag{12}$$

3. Solution of the system of wave equations on the basis of the above-mentioned theorem in the form of the dependence of pressure on a set of monopoles and dipoles for shortwave acoustic emitters is determined by the Helmholtz–Brazhnikov integral [29]:

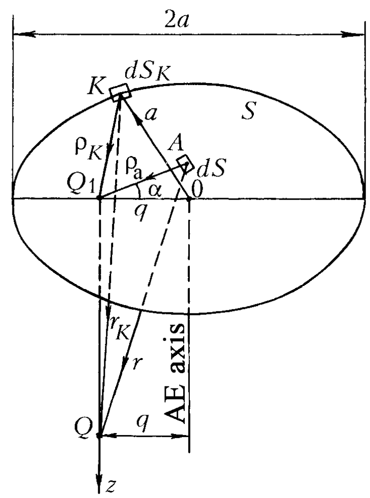

Fig. 1. Schematic of calculation of the parameters of the emitter field.

$$P = -\frac{1}{4\pi} \iint_S P_s r^{-1} \frac{\partial}{\partial r} [(r + z)\,\psi]\, dS\,. \tag{13}$$

Using the coordinates mentioned above and integrating (13) with respect to ψ from (1) and P_s from (9), we obtain the Brazhnikov formula for pressure on the axis of the acoustic emitter:

$$P\,(z) = P_0 \left[1 - \frac{1}{2} \left(1 + \frac{z}{r_m} \right) \exp\,(-\,i\varphi_0) \right] \exp\, i\,(\omega t - kz)\,. \tag{14}$$

The axial pressure in the near-field zone of the acoustic emitter depending on the relative distance z/a experiences a number of amplitude extrema with the phase φ_0 a multiple of π. The computation parameter z/r_m, according to (4), is related to z/a as

$$\frac{z}{r_m} = \frac{z}{a} \left[1 + \left(\frac{z}{a} \right)^2 \right]^{-0.5}.$$

In the calculation of extrema (phase φ_0 is a multiple of π for them), the distances z/a from the surface of the acoustic emitter with the wave parameter ka, at which they lie, are determined from Eq. (7) written in the form

$$\frac{\varphi_0}{ka} = \frac{r_m - z}{a} = (1 - z^2/a^2)^{0.5} - z/a \quad \text{or} \quad \left(\frac{\varphi_0}{ka} + \frac{z}{a} \right)^2 = 1 + \frac{z^2}{a^2}\,,$$

whence

$$z/a = 0.5 \left(\frac{ka}{\varphi_0} - \frac{\varphi_0}{ka} \right).$$

To determine pressure in the paraxial domain ($0 < q < 0.2a$) of the field we place the origin of the system of coordinates (z, ρ_a, α) in the projection Q_1 of the point of receipt (lying at a distance q from the axis of the acoustic emitter) on the plane of the acoustic emitter (see Fig. 1). The parameter ρ_a varies from 0 to ρ_K — a value which depends on q and is determined by the distance from the center Q_1 of the system of coordinates to the point K at the center of the element dS_K sliding along the end of the acoustic emitter. The integration variable r changes here from z to

TABLE 1. Dependences of the Modulus of the Pressure Ratio $|P/P_0|$ for the Acoustic Emitter with ka = 16 on the Axis (q = 0) and in the Paraxial Domain (q = 0.1a) of the Field on the Relative Distance z/a by the Formulas of Concepts 1, 2, and 3

z/a	φ_0/π	$q = 0$			$q = 0.1a$	
		1	2	3	1	2
		Shtensel' (6)	Skudrzyk (12)	Brazhnikov (14)	Shokh (8)	(18)
0.018	5	2	1.018	1.509		
0.244	4	0	0.764	0.382	0.519	0.702
0.554	3	2	1.484	1.742	1.567	1.421
1.077	2	0	0.268	0.134	0.275	0.372
2.448	1	2	1.926	1.963	1.910	1.877
4	0.627	1.666	1.742	1.654	1.635	1.623
6	0.421	1.230	1.220	1.225	1.219	1.215
8	0.317	0.955	0.953	0.954	0.950	0.949

$$r_K = (z^2 + \rho_K^2)^{0.5}, \quad (15)$$

and the variable α — from $-\pi$ to $+\pi$. Taking into account the values of ψ from (1), dS from (11), and the limits of variation of r and α, integral (13) takes on the form

$$P_q = -\frac{1}{4\pi}\int_{-\pi}^{\pi} P_s \psi\,(r+z)\Big|_z^{r_K} d\alpha = \frac{P_s}{2\pi}\int_0^{\pi}\left[2\exp(-ikz) - \left(1+\frac{z}{r_K}\right)\exp(-ikr_K)\right] d\alpha \quad (16)$$

or

$$P_q = P_s \exp(-ikz) - P_d,$$

where the first term can be called the pressure of a plane wave of *direct radiation*, and the second term — diffraction pressure interfering with *direct radiation*:

$$P_d = \frac{P_s}{2\pi}\int_0^{\pi}\left(1+\frac{z}{r_K}\right)\exp(-ikr_K)\,d\alpha. \quad (17)$$

For a vector triangle with sides q, a, and ρ_K (see Fig. 1), we have the equation $\rho^2 + 2q\rho_K\cos\alpha + q^2 - a^2 = 0$ the solution of which is $\rho_K = (q^2\cos^2\alpha + a^2 - q^2)^{0.5} - q\cos\alpha$. Hence, for the paraxial domain (taking into account $q^2 << a^2$), $\rho_K = a - q\cos\alpha$ and, according to (15),

$$r_K = (z^2 + a^2 + q^2 - 2qa\cos\alpha)^{0.5} = r_m(1 - 2qar_m^{-2}\cos\alpha)^{0.5}.$$

Restricting ourselves in the expansion into series of r_K and z/r_K to two terms, we obtain for the exponent index in (17): $kr_K = kr_m - kqar_m^{-1}\cos\alpha$ and for the characteristic of the direction of the "slide" dipole: $z/r_K = r_m^{-1} z/r_m + zqar_m^{-3}\cos\alpha$. With account for this, expression (17) takes on the form

$$P_d = \frac{P_s}{2\pi}\exp(-ikr_m)\int_0^{\pi}\left(1+\frac{z}{r_m} + zqar_m^{-3}\cos\alpha\right)\exp(i\varepsilon_q\cos\alpha)\,d\alpha,$$

where r_m and ε_q are determined from (4) and (9).

It is known that

$$\int_0^{\pi} \exp(i\varepsilon \cos\alpha)\, d\alpha = \pi J_0(\varepsilon)\,, \quad \int_0^{\pi} \exp(i\varepsilon \cos\alpha) \cos\alpha d\alpha = i\pi J_1(\varepsilon)\,.$$

Consequently, the diffraction pressure is

$$P_{\mathrm{d}} = P_{\mathrm{s}} \left[\frac{z + r_{\mathrm{m}}}{2r_{\mathrm{m}}} J_0(\varepsilon_q) + \frac{zqa}{2r_{\mathrm{m}}^3} J_1(\varepsilon_q) \right] \exp(-ikr_{\mathrm{m}})\,.$$

Neglecting here the second term as a value of second order of smallness, according to (9) and (16) we obtain an expression for total pressure in the paraxial domain of the field at the point lying at a distance q from the axis of the acoustic emitter:

$$P_q = P_0 \left[1 - \frac{z + r_{\mathrm{m}}}{2r_{\mathrm{m}}} J_0(\varepsilon_q) \exp(-i\varphi_0) \right] \exp i(\omega t - kz)\,. \tag{18}$$

The results of the calculation of extrema of the moduli of axial pressure $|P(z)|/P_0$ and pressure $|P_q|/P_0$ in the paraxial domain in the function z/a by the formulas of the mentioned concepts 1, 2, and 3 within the near-field zone of the acoustic emitter with $ka = 16$ are given in Table 1.

Maxima of axial pressure appear at φ_0 equal to an odd π number, with the end of the zone being correspondent to the maximum (at $\varphi_0 = \pi$) which is most distant from the indicated acoustic emitter. The maximum (at $\varphi_0 = 5\pi$) closest to the acoustic emitter lies at a distance of $0.02a$ from it.

Pressure minima appear at the phase φ_0 equal to an even π number. The farthest removed minimum (at $\varphi_0 = 2\pi$) lies at a distance of $1.08a$ from the acoustic emitter of the given type ($ka = 16$) and the closest one — at a distance of $0.24a$.

According to concept 1 [Rayleigh integral, the Shtensel' formula (6), and the Shokh formula (8)], all minima of axial pressure are zero and maxima are $2P_0$. This is inconsistent with the experimental data. The calculations of pressure in the paraxial domain by formula (8) give overestimated values near axial maxima and underestimated ones near axial minima.

By concept 3 [integral (13), the Brazhnikov formula (14), and revised formula (18)], pressure minima on the axis of the acoustic emitter do not take a zero value with increase in z/a. They fall from $0.38P_0$ at $\varphi_0 = 4\pi$ to $0.13P_0$ at $\varphi_0 = 2\pi$. As z/a increases, the maxima increase from $1.51P_0$ at $\varphi_0 = 5\pi$ near the acoustic emitter to $1.96P_0$ at $\varphi_0 = \pi$ at the end of the near-field zone. Here, the calculations are in agreement with the experimental data of [30] at a frequency of 350 kHz ($ka = 59$) and with our studies of the liquid sound duct of the phase meter of flow velocity at a frequency of 500 kHz ($ka = 16$) in the case of comparison by a generalized coordinate $z\lambda a^{-2}$.

It follows from the calculations by (18) that at the side of axial ($q = 0$) maxima at z/a equal to 0.554 and 2.448 (end of the near-field zone), pressure at $q = 0.1a$ decreases toward a side (annular) maximum by 18 and 4%, respectively. At the side of axial minima (at $z/a = 0.244$ and 1.077), pressure at $q = 1.0a$ increases toward a side (annular) maximum by 1.8 and 2.8 times, respectively.

According to concept 2 [integral (10) and the Skudrzyk formula (12)], the pressure minima are higher than the minima of concept 3 and the maxima are lower than the maxima of this concept.

The expressions of concepts 1 and 2 are simplified solutions of the wave equations. This determination could be put into question if the expressions for axial pressures obtained from the same system of wave equations by (12) of concept 2 and by (6) of concept 3 were equal to each other at any z/a. In fact, they are different, which is determined by the expression

$$P(z)_2 - P(z)_1 = P_0 \left(1 - \frac{z}{r_{\mathrm{m}}} \right) \exp i(\omega t - kr_{\mathrm{m}})\,.$$

The difference of moduli of these pressures has a value which decreases in amplitude (for $ka = 16$) from $0.98P_0$ near the acoustic emitter to $0.07P_0$ at the end of the near-field zone.

In the near-field zone of the paraxial domain, the pressure is determined inaccurately by the Shokh formula, thus approaching the values obtained by the revised formula (18) beyond this zone.

Consequently, the integral and formulas of concept 3 can be assumed to be complete solutions of the system of wave equations for pressure P_q at the point Q of the field and $P(z)$ on the axis of a short-wave acoustic emitter. Therefore, employment of concepts 1 and 2 is admissible in the far-field zone and concept 3 — in all zones of the field of the acoustic emitter.

NOTATION

a, radius of the acoustic emitter, m; c, velocity of propagation of ultrasound, m/sec; dS, element of the surface of the acoustic emitter, m^2; J_0 and J_1, Bessel functions of zero and first order; k, wave-number, m^{-1}; ka, wave parameter of the acoustic emitter; P and $P(z)$, ultrasonic pressure and pressure on the axis of the acoustic emitter, N/m^2; P_0 and P_s, pressure of the radiated wave and that at the point of radiation A (see Fig. 1), N/m^2; q, distance from the point of ultrasound arrival Q to the axis of the acoustic emitter, m; r, distance between the points A and Q, m; r_m, distance from the point Q on the axis to the end of the acoustic emitter, m; S, emitter surface, m^2; v_n, normal component of vibrational velocity of the acoustic emitter, m/sec; t, time, sec; z, coordinate, projection on the axis of the acoustic emitter, m; α, angular coordinate, rad; ω, angular frequency, sec^{-1}; ε_q, argument of the Bessel function; ρ, medium density, kg/m^3; ρ_a, radius-vector at the point of radiation, m; φ and φ_s, elastic potential of the field and potential of vibrational velocity of the radiation surface S, m^2/sec; φ_0, phase of the admitted wave on the axis of the acoustic emitter, rad; ψ, potential of the point source, m. Subscripts: a, acoustic; m, maximum; n, normal component; s, surface; d, diffraction.

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